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# 3–5 Equations with the Variable on Both Sides

**Objective:** To solve equations with the variable on both sides.

***Example 1*** Solve $5x = 2x + 15$.

***Solution***

$$\begin{aligned} 5x - 2x &= 2x + 15 - 2x \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Subtract $2x$ from each side.

*Check:* $5(5) \stackrel{?}{=} 2(5) + 15$

$25 \stackrel{?}{=} 10 + 15$

$25 = 25\ \surd$

The solution set is $\{5\}$.

***Example 2*** Solve $4x = 30 - x$.

***Solution***

$$\begin{aligned} 4x + x &= 30 - x + x \\ 5x &= 30 \\ x &= 6 \end{aligned}$$

Add $x$ to each side.

The solution set is $\{6\}$.

**Solve.**

**1.** $5n = 3n + 8$ **2.** $7a = 2a + 30$ **3.** $y = 20 - 3y$ **4.** $3b = 80 - 5b$

**5.** $10n = 36 - 2n$ **6.** $2x = 20 - 8x$ **7.** $21a = 56 + 7a$ **8.** $30 + 6x = 12x$

**9.** $-9a = -12a - 45$ **10.** $33c + 60 = 21c$ **11.** $72 - 4n = -22n$ **12.** $-11a = -12a - 21$

***Example 3*** Solve $\frac{2}{5}x + 12 = x$.

***Solution***

$$\begin{aligned} \tfrac{2}{5}x + 12 - \tfrac{2}{5}x &= x - \tfrac{2}{5}x && \text{Subtract } \tfrac{2}{5}x \text{ from each side.} \\ 12 &= \tfrac{5}{5}x - \tfrac{2}{5}x && \text{Rewrite } 1x \text{ as } \tfrac{5}{5}x. \\ 12 &= \tfrac{3}{5}x \\ \tfrac{5}{3} \cdot \tfrac{12}{1} &= \tfrac{5}{3}\left(\tfrac{3}{5}x\right) && \text{Multiply each side by } \tfrac{5}{3}, \text{ the reciprocal of } \tfrac{3}{5}. \\ 20 &= x && \text{The solution set is } \{20\}. \end{aligned}$$

***Example 4*** Solve $\frac{6 + x}{3} = x$.

***Solution***

$$\begin{aligned} 3\left(\tfrac{6 + x}{3}\right) &= 3 \cdot x && \text{Multiply each side by 3, the reciprocal of } \tfrac{1}{3}. \\ 6 + x &= 3x \\ 6 + x - x &= 3x - x && \text{Subtract } x \text{ from each side.} \\ 6 &= 2x \\ 3 &= x && \text{The solution set is } \{3\}. \end{aligned}$$

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### 3–5 *Equations with the Variable on Both Sides* *(continued)*

**Solve.**

**13.** $\frac{2}{3}x - 5 = x$ **14.** $\frac{3}{4}x - 8 = x$ **15.** $x = \frac{1}{2}x + 7$ **16.** $x = \frac{4}{5}x - 9$

**17.** $\frac{x - 2}{3} = x$ **18.** $\frac{3 + y}{4} = y$ **19.** $y = \frac{7 - 2y}{5}$ **20.** $x = \frac{9 + x}{4}$

**Vocabulary**

**Empty set or null set** The set with no members.

**Identity** An equation that is true for every value of the variable(s).

**Symbol** $\phi$ (empty set, or the null set)

**CAUTION** An equation may have no solution, or it may be satisfied by every real number.

***Example 5*** Solve: **a.** $5(a - 2) - 3 = 3a + 7 + 2a$ **b.** $\frac{1}{3}(24x - 15) = 8x - 5$

***Solution***

**a.** $5a - 10 - 3 = 5a + 7$

$5a - 13 = 5a + 7$

$-13 = 7$ ← **False**

The equation has *no solution.*

**b.** $8x - 5 = 8x - 5$ ← **Identity**

An identity is true for every value of the variable.

The solution set is {real numbers}.

**Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.**

**21.** $2(x - 3) = 5x$ **22.** $4(y - 5) = 9y$ **23.** $3n = 6(3 - n)$

**24.** $-3m = 5(2 - m)$ **25.** $2(a - 1) = 2a + 3$ **26.** $\frac{1}{4}(28x - 8) = 7x - 2$

**27.** $\frac{1}{3}(3x - 3) + 2 = 2x$ **28.** $4(a - 1) - 5 = 3a + 7$ **29.** $3(5 + y) - y = 2y + 15$

**30.** $4a + 7 + a = 3(a - 1)$ **31.** $\frac{3n - 15}{4} = 2n$ **32.** $\frac{2n - 9}{2} = n$

## Mixed Review Exercises

**Simplify.**

**1.** $3 + \left(-\frac{1}{3}\right) + \left(-\frac{5}{3}\right)$ **2.** $-2\frac{3}{4} + 1\frac{1}{4}$ **3.** $-115 - (-10)$

**4.** $15x + (-3x) - 2$ **5.** $-4y + 5 + 18y + 23$ **6.** $6(-2)(-5)(-4)$

**Solve.**

**7.** $-2 - x = 5$ **8.** $4 + (1 + k) = 2$ **9.** $3x = -276$

**10.** $\frac{1}{2}x = 3\frac{1}{2}$ **11.** $\frac{x}{6} = 7$ **12.** $-10\frac{2}{3} = -\frac{1}{3}x$